MATH 270
SUMMER 2007
HOMEWORK 2

Due Tuesday June 19, 2007.

1. Let x, y, z be real numbers. Prove the following.
   a) (3 pt) (*The Triangle Inequality.*) $|x + y| \leq |x| + |y|$.
   b) (3 pt) Show that if $x \leq y$ and $z < 0$ then $zx \geq zy$ and $\frac{x}{z} \geq \frac{y}{z}$.

2. (5 pt) Let $\epsilon, \alpha > 0$ be a positive real numbers. Show that there exists a natural number N such that $N\epsilon > \alpha$.

3. Let $\alpha < \beta$ be real numbers and $n \in \mathbb{N}$.
   a) (5 pt) Show that if $\beta - \alpha > n$ then there are at least n distinct integers strictly between α and β.
   b) (5 pt) Show that if $\alpha < \beta$ are real numbers then there is a rational number q such that $\alpha < q < \beta$.

4. Let n be an integer and $q \in \mathbb{Q}$.
   a) (3 pt) Show that if $q^2 \in \mathbb{Z}$ then $q \in \mathbb{Z}$.
   b) (3 pt) Show that n^2 is odd if and only if n is odd.
   c) (3 pt) Show that $\sqrt{2}$ is irrational.