MATH 270
SPRING 2003
HOMEWORK 3

Due Wednesday February 5, 2003.

1. (3 pt) Show that the statement $P \Longrightarrow Q$ is equivalent to $\sim Q \Longrightarrow \sim P$.

2. Let $n, m \in \mathbb{Z}$ be integers (not both 0), and let $d = \gcd(n, m)$ and $L = \text{lcm}(n, m)$.
   (a) (5 pt) Show that there exist $a, b \in \mathbb{Z}$ such that $an + bm = d$ (so the greatest common divisor of two integers is a linear combination of the two elements...this is a very important property of the integers).
   (b) (5 pt) Show that there exists $x, y \in \mathbb{Z}$ such that $xn + ym = L$.
   (c) (5 pt) Show that if $\gcd(n, m) = d$ and $\text{lcm}(n, m) = L$, then $nm = dL$.

3. For the following statements, give the negation.
   (a) (3 pt) Bill and Fred went to see the movie.
   (b) (3 pt) The equation $f(x) = 0$ has a unique solution in $\mathbb{R}$.
   (c) (3 pt) Every differentiable function is continuous.

4. (10 pt) Let $\{A_\alpha\}_{\alpha \in \Lambda}$ be a collection of sets all contained in some universal set U. For any subset $B \subseteq U$ we denote the compliment of B in U by B^{c}. Show the following.
   (a) $(\bigcup_{\alpha \in \Lambda} A_\alpha)^{\text{c}} = \bigcap_\alpha (A_\alpha)^{\text{c}}$.
   (b) $(\bigcap_{\alpha \in \Lambda} A_\alpha)^{\text{c}} = \bigcup_\alpha (A_\alpha)^{\text{c}}$.