MATH 270
SUMMER 2004
HOMEWORK 3

Due Monday June 28, 2004.

1. Let A, B, and C be sets. Prove the following.
   a) (3 pt) $A \setminus (B \bigcup C) = (A \setminus B) \bigcap (A \setminus C)$.
   b) (3 pt) $A \setminus (B \bigcap C) = (A \setminus B) \bigcup (A \setminus C)$.

2. We recall that if $m, n \in \mathbb{N}$ then the *greatest common divisor* of m and n (we write $\gcd(m, n) = d$ or $(m, n) = d$) is a (positive) common divisor of m and n with the property that if d' is another common divisor of m and n then $d'|d$. Additionally the *least commmon multiple* of m and n (we write $\text{lcm}(m, n) = L$) is a positive common multiple of m and n such that if L' is another common multiple of m and n then $L|L'$.

   Prove the following.
   a) (5 pt) If $m, n \in \mathbb{N}$ then $\text{lcm}(m, n)$ exists.
   b) (5 pt) If $m, n \in \mathbb{N}$ and $d = \gcd(m, n)$ then there exist $r, s \in \mathbb{Z}$ such that $rm + sn = d$. Your proof should demonstrate the existence of the greatest common divisor of m and n. (This is a very important property of the integers.)
   c) (5 pt) If $m, n \in \mathbb{N}$, $d = \gcd(m, n)$, and $L = \text{lcm}(m, n)$, then $mn = dL$.

3. Let A, B and C be sets and denote the power set of the set X by $P(X)$.
   a) (3 pt) Show that $C \subseteq A \bigcap B$ if and only if $C \subseteq A$ and $C \subseteq B$.
   b) (5 pt) Show that $P(A \bigcap B) = P(A) \bigcap P(B)$.
   c) (5 pt) Show that $P(A) \bigcup P(B) \subseteq P(A \bigcup B)$. When does set equality hold here?