MATH 270
SPRING 2003
HOMEWORK 4

Due Friday February 14, 2003.

1. (3 pt) Let A and B be sets. Show that $(A \setminus B) \bigcup (B \setminus A) = (A \bigcup B) \setminus (A \bigcap B)$.

2. (10 pt) Let A, B, and C be sets. Show that $(A \setminus B) \setminus C = A \setminus (B \setminus C)$ if and only if $A \bigcap C = \emptyset$.

3. Consider the collection of sets $A_n = (-\frac{1}{n}, \frac{1}{n}), n \in \mathbb{N}$. Let $I \subseteq \mathbb{N}$ be a nonempty subset of $\mathbb{N}$. Show the following.

   (a) (5 pt) $\bigcup_{m \in I} A_m = A_k$ for some $k \in I$.
   (b) (5 pt) $\bigcap_{m \in I} A_m = A_k$ for some $k \in I$ if and only if I is finite.
   (c) (5 pt) $\bigcap_{m \in I} A_m$ is a set containing a single real number (what is it?) if and only if I is infinite.

4. We say that a collection of sets $\{A_n \mid n \in \mathbb{N}\}$ is *nested* if for all $i, j \in \mathbb{N}$ with $i \leq j$, $A_j \subseteq A_i$ (for example, the sets in the previous problem are nested).

   (a) (3 pt) Give an example of a nested collection of open intervals $(a_i, b_i) \subseteq \mathbb{R}$ such that $\bigcap_i (a_i, b_i) = [a, b]$ (and prove that your example works).
   (a) (3 pt) Give an example of a collection of closed intervals $[a_i, b_i] \subseteq \mathbb{R}$ such that $\bigcup_i [a_i, b_i] = (a, b)$ (and prove that your example works).