MATH 270
SUMMER 2004
HOMEWORK 5

Due Wednesday July 7, 2004.

1. Suppose that p is a prime number and $n, m \in \mathbb{N}$.
   a) (5 pt) Show that if $a, b \in \mathbb{N}$ and $p|ab$ then $p|a$ or $p|b$.
   b) (5 pt) Show that if $1 \leq m \leq p-1$ then $p|\binom{p}{m}$. Is this result true if the assumption that p is a prime is omitted?
   c) (5 pt) Show that $n^p - n$ is always divisible by p.

2. Suppose that there are 10 finalists in a small lottery.
   a) (3 pt) If there are three $100 prizes, two $200 prizes, and a $300 prize to be given out, how many ways are there to distribute the money?
   b) (3 pt) If there are five prizes of $200 to be given, how many ways are there to distribute the money?

3. Let S be a nonempty partially ordered set and $T \subseteq S$ a subset.
   a) (5 pt) Show that T is also a partially ordered set.
   b) (5 pt) Show that if S is totally ordered then so is T.
   c) (5 pt) Is the converse to part b) true?
   d) (5 pt) Show that if S is finite and $x \in S$, then x is contained in a maximal totally ordered subset of S (by maximal totally ordered subset, I mean a subset $T \subseteq S$ that is totally ordered such that there is no other totally ordered subset of S *properly* containing T).

4. (3 pt) Let A and B be finite sets. How many relations are there from A to B? Which would be worse, me asking you to list the relations from $\{1, 2, 3, 4, 5, 6, 7\}$ to $\{a, b, c, d, e\}$, or me asking you to list the relations on $\{u, v, w, x, y, z\}$?