MATH 270
SPRING 2003
HOMEWORK 6

Due Friday March 7, 2003.

1. (3 pt) We say that $p \in \mathbb{N}$ is *prime* if the only divisors of p (in $\mathbb{N}$) are itself and 1. Suppose that $p \in \mathbb{N}$ is prime and $n \in \mathbb{N}$ is a natural number. Show that if $m \in \mathbb{N}$ is a natural number such that $\gcd(m, p) = 1$ and m divides pn then m divides n. (Hint: perhaps an earlier homework will be useful).

2. (5 pt) Use the previous problem to show that if p is a prime and $a, b \in \mathbb{N}$ are such that p divides ab, then p must divide either a or b.

3. (3 pt) Use the previous result to show that if p is a prime and n is a natural number such that $1 \leq n \leq p - 1$ then p divides the binomial coefficient $\binom{p}{n}$.

4. (3 pt) Use the previous results to show that if p is prime and $n \in \mathbb{N}$, then $n^p - n$ is a multiple of p.

5. (10 pt) Let A be a set of n elements ($n \geq 1$). Find the number of distinct equivalence relations that can be imposed on A for $n = 5, 6$.