1. Let \( f : A \rightarrow B \) and \( g : B \rightarrow C \) be functions.
   a) (3 pt) Show that if \( g \circ f \) is onto, then \( g \) is onto.
   b) (3 pt) Show that if \( g \circ f \) is one to one, then \( f \) is one to one.
   c) (3 pt) Give an example where \( g \circ f \) is onto, but \( f \) is not onto.
   d) (3 pt) Give an example where \( g \circ f \) is one to one, but \( g \) is not one to one.
   e) (3 pt) Show that if \( f \) and \( g \) are both one to one, then so is \( g \circ f \).
   f) (3 pt) Show that if \( f \) and \( g \) are both onto, then so is \( g \circ f \).

2. (5 pt) Show that if \( f : A \rightarrow B \) is both one to one and onto then there is a function
   \( g : B \rightarrow A \) such that \( f \circ g = I_B \) and \( g \circ f = I_A \).

3. In this problem, we count equivalence relations and partial orderings.
   a) (3 pt) How many distinct equivalence relations can be placed on a set of 5
      elements?
   b) (3 pt) How many linear orderings can be placed on a set with \( n \) elements?
   c) (3 pt) How many distinct partial orderings can be placed on a set with 5
      elements?

4. Let \( f : A \rightarrow B \) be a function and let \( \{S_k\}_{k \in I} \) be a collection of subsets of \( A \)
   (indexed by some set \( I \)).
   a) (5 pt) Show that if \( f \) is one to one, then \( f(\bigcap_{k \in I} S_k) = \bigcap_{k \in I} f(S_k) \).
   b) (5 pt) Given an example where \( f(\bigcap_{k \in I} S_k) \neq \bigcap_{k \in I} f(S_k) \).

5. Let \( A \) and \( B \) be finite sets, each with \( n \) elements, and let \( f : A \rightarrow B \) be a function.
   a) (5 pt) Show that \( f \) is one to one if and only if \( f \) is onto.
   b) (5 pt) Show that if \( A \) and \( B \) are infinite then neither implication from part a)
      necessarily holds.