MATH 270
SUMMER 2007
HOMEWORK 7

Due Friday July 13, 2007.

1. Let $f : A \longrightarrow B$ and $g : B \longrightarrow C$ be functions.
   a) (3 pt) Show that if $g \circ f$ is onto, then g is onto.
   b) (3 pt) Show that if $g \circ f$ is one to one, then f is one to one.
   c) (3 pt) Give an example where $g \circ f$ is onto, but f is not onto.
   d) (3 pt) Give an example where $g \circ f$ is one to one, but g is not one to one.
   e) (3 pt) Show that if f and g are both one to one, then so is $g \circ f$.
   f) (3 pt) Show that if f and g are both onto, then so is $g \circ f$.

2. (5 pt) Show that if $f : A \longrightarrow B$ is both one to one and onto then there is a function $g : B \longrightarrow A$ such that $f \circ g = I_B$ and $g \circ f = I_A$.

3. In this problem, we count equivalence relations and partial orderings.
   a) (3 pt) How many distinct equivalence relations can be placed on a set of 5 elements?
   b) (3 pt) How many linear orderings can be placed on a set with n elements?
   c) (3 pt) How many distinct partial orderings can be placed on a set with 5 elements?

4. Let $f : A \longrightarrow B$ be a function and let $\{S_k\}_{k \in I}$ be a collection of subsets of A (indexed by some set I).
   a) (5 pt) Show that if f is one to one, then $f(\bigcap_{k \in I} S_k) = \bigcap_{k \in I} f(S_k)$.
   b) (5 pt) Given an example where $f(\bigcap_{k \in I} S_k) \neq \bigcap_{k \in I} f(S_k)$.

5. Let A and B be finite sets, each with n elements, and let $f : A \longrightarrow B$ be a function.
   a) (5 pt) Show that f is one to one if and only if f is onto.
   b) (5 pt) Show that if A and B are infinite then neither implication from part a) necessarily holds.