MATH 270
SPRING 2003
HOMEWORK 9

Due Wednesday April 23, 2003.

1. Let G be a group. Prove the following.
   a) (3 pt) The identity element of G is unique.
   b) (3 pt) If $x \in G$ then the inverse of x is unique.

2. (5 pt) Let G be a group that has the property that every element of G has order no more than 2. Show that G is necessarily an abelian group.

3. (3 pt) Let G be a finite group and let $x \in G$. Show that the order of x is finite.

4. (6 pt) Let p be a positive prime integer. Give an example of a group of order p^2 which has an element of order p^2 and give an example of a group of order p^2 that has no element of order p^2.

5. (10 pt) Let A be any nonempty set and let $S(A) = \{f : A \longrightarrow A |\ f \text{ is bijective}\}$. Show that $S(A)$ forms a group under function composition. If $|A| = n$ then we often write $S(A) = S_n$. Find the order of S_n.

6. (10 pt) Let $GL_2(\mathbb{R})$ be the set of 2×2 matrices with real entries and nonzero determinant. Let $SL_2(\mathbb{R})$ be the set of 2×2 matrices with real entries and determinant 1. Show that $GL_2(\mathbb{R})$ is a group containing $SL_2(\mathbb{R})$ as a subgroup.