MATH 270
SUMMER 2004
EXAM 1

1. (10 pt) Prove that for all $n \in \mathbb{N}$

$$\prod_{k=1}^{n}(2k-1) = 1 \cdot 3 \cdot 5 \cdot \ldots \cdot (2n-1) = \frac{(2n)!}{n!2^n}.$$

2. (10 pt) Use induction to show that if S is a finite set with n elements then $|P(S)| = 2^n$.

3. Consider the following statements:
   (1) "Bill or Sam went to the movies."
   (2) "If my name is Weaselnose, then Britney Spears needs some penicillin."
   a) (5 pt) Negate the first statement (and please be clear).
   b) (5 pt) State the contrapositive of the second statement.
   c) (5 pt) For the second statement, consider the fact that my name is "Jim". Does it logically follow that Miss Spears is disease-free? Explain why or why not.

4. Poker is a card game played from a deck of 52 cards. In this deck there are four suits (hearts, spades, clubs and diamonds) consisting of 13 cards apiece (2,3,4,5,6,7,8,9,10,J,Q,K,A). A poker hand consists of 5 cards.
   a) (5 pt) How many 5 card hands are there?
   b) (5 pt) How many hands are there that consist of 4 like cards (e.g. all 2's and an extra, the so-called "four of a kind")?
   c) (5 pt) How many hands are there that consist of precisely 3 like cards AND 2 like cards (a "full house")?