MATH 270
SPRING 2003
EXAM 1
IN CLASS PORTION

1. (15 pt) Prove that for every $n \in \mathbb{N}$, $n^3 - n$ is divisible by 3.

2. (15 pt) Suppose that you have a square and you wish to label each of the corners with the numbers $1, 2, 3, 4$. We will say that two such labellings L_1 and L_2 are equivalent ($L_1 \sim L_2$) if L_2 is a looks like L_1 after a rotation.
    a) How many labellings are possible?
    b) Show that $\sim$ is an equivalence relation.
    c) How many distinct equivalence classes are there (modulo $\sim$)?

3. (10 pt) Suppose that you have three sets A, B, C with $13, 12$ and 24 elements respectively. You also know that the number of elements in $A \bigcup B \bigcup C$ is 37 and that $|A \bigcap B| = 3$, $|B \bigcap C| = 4$, and $|A \bigcap C| = 6$. Find the number of elements in $A \bigcap B \bigcap C$.

4. (10 pt) Perform the following operations:
    a) Negate the statement: "Either Bill and Joe went to the movies or Cletus blew up a gopher".
    b) Give the contrapositive to the statement: "A natural number is prime if it is only divisible by itself and 1".