MATH 270
SPRING 2003
EXAM 1
TAKE HOME PORTION

Due Wednesday February 26, 2003.

Instructions: You may use notes and your book, but I am the only biological resource that you should utilize. This portion of the exam will be weighted equally with the in class portion.

1. Consider the 5×5 Rubik's cube.

   a) (5 pt) If the cube were pulled apart, and reassembled, how many arrangements are possible (pay attention only to the blocks that are visible)? (Hint: it would be good to draw a picture to help you out).

   b) (5 pt) Explain clearly why the number that you get in part a) must be greater than the number of arrangements obtained by only rotaing the cube (that is, not pulling it apart).

2. Poker is a card game where each player receives 5 cards and hopes for the "highest" hand. In poker, three of a kind (three cards of the same value), beats two pairs (two distinct pairs of cards with the same value), and two pairs beats a single pair. The game is optionally played with some number of "jokers" (wild cards that can be declared any value that the holder desires).

   a) (5 pt) How many distinct hands are possible in poker (no jokers)?

   b) (5 pt) How many distinct single pair hands are there (no jokers)?

   c) (5 pt) How many distinct two pair hands are there (no jokers)?

   d) (5 pt) How many distinct three of a kind hands are there (no jokers)?

   e) (5 pt) How many distinct three of a kind hands are there (with one joker)?

   f) (5 pt) Show/explain why if even one joker is introduced to the deck, you should never bet on a two pairs hand.

For your interest you should be careful about what you read. In particular, I found a site called About-Poker (http://www.about-poker.com/pokerodds.htm) where a number of their computations are sloppy or just plain wrong.

3. Let $n \geq 0$ be a fixed integer. We define a relation on $\mathbb{Z}$ by saying that two integers $k, m \in \mathbb{Z}$ are equivalent ($k \sim_n m$) if $k - m = rn$ for some $r \in \mathbb{Z}$.

   a) (5 pt) Show that $\sim_n$ is an equivalence relation on $\mathbb{Z}$.

   b) (5 pt) Find the number of *distinct* equivalence classes in $\mathbb{Z}/\sim_n$ for all $n \geq 0$.