MATH 270
SPRING 2003
EXAM 2
IN CLASS PORTION

1. (5 pt) Give an example of a partially ordered set that is not totally ordered (and explain why your example works).

2. (7 pt) Let $\mathbb{R}$ denote the set of real numbers, $\wp$ the set of nonzero prime integers, E the set of even integers, $\mathbb{Q}$ the rational numbers, D the set of integers that are not divisible by any prime, M the number of molecules in the galaxy, and $\mathbb{N}$ the natural numbers. Arrange the sets $\mathbb{R}, \wp, E, \mathbb{Q}, D, M$, and $\mathbb{N}$ in increasing order of size (use the notations $<$ and $=$ only).

3. (5 pt) Suppose that you have a function f from a set to itself ($f : A \longrightarrow A$) and suppose that f is one to one. Is it necessarily onto? Prove or give a counterexample.

4. Consider the functions $f : A \longrightarrow B$ and $g : B \longrightarrow C$.
   a) (5 pt) Show that if $g \circ f$ is onto then g is onto.
   b) (5 pt) Show that if $g \circ f$ is one to one then f is one to one.

5. (5 pt) Let $f : A \longrightarrow B$ be a function. Show that f is one to one if and only if for all $b \in B$, $|f^{-1}(\{b\})| \leq 1$.

6. (5 pt) Let $f : A \longrightarrow B$ be a one to one function. Show that there is an onto function $g : B \longrightarrow A$. (*Note: it is also true that if $f : A \longrightarrow B$ is onto then there is a function $g : B \longrightarrow A$ that is one to one, but I had to use the Axiom of Choice...see if you can prove this for extra credit.*)