MATH 270
SUMMER 2004
EXAM 2
IN CLASS PORTION

1. Let $f : A \longrightarrow B$ be a function.
   a) (5 pt) Show that f is one to one if and only if $|f^{-1}(b)| \leq 1$ for all $b \in B$.
   b) (5 pt) Show that f is onto if and only if $|f^{-1}(b)| \geq 1$ for all $b \in B$.

2. (5 pt) Let A and B be finite sets with $|A| = |B|$ and $f : A \longrightarrow B$ a function. Show that f is one to one if and only if f is onto.

3. Let S be a finite set with n elements.
   a) (5 pt) How many functions are there from S to S?
   b) (5 pt) How many bijections are there from S to S?

4. Consider a standard deck, D, of 52 playing cards. We declare that two cards, $x, y \in D$ are equivalent ($x \sim y$) if and only if the suit of x and the suit of y are the same.
   a) (5 pt) Show that $\sim$ is an equivalence relation on D.
   b) (5 pt) How many equivalence classes are there (and find a representative for each class)?
   c) (5 pt) How many elements are there in each of the equivalence classes that you found?

5. (5 pt) Order the following sets (use only the symbols "$<$" and "$=$"). Briefly explain your answer.
   a) The set, $\mathbb{N}$, of natural numbers.
   b) The set, $\mathbb{R}$, of real numbers.
   c) The set, $\mathfrak{C}$, of functions $f : \mathbb{R} \longrightarrow \mathbb{R}$.
   d) The set, $\mathbb{Q}$, of rational numbers.
   e) The set, P, of distinct poker hands in a standard deck of 52 cards.
   f) The set, J, of good Jennifer Lopez movies.
   g) $P(\mathbb{N})$, the power set of the natural numbers.