MATH 270
SUMMER 2004
EXAM 2
TAKE HOME PORTION

Due at the beginning of the in-class exam on Tuesday, July 20, 2004.

1. Let $a, b, c, d \in \mathbb{R}$ with $a \neq b$ and $c \neq d$ (the notation $|(a,b)|$ refers to the cardinality of the interval (a,b)). Show the following.

   a) (5 pt) $|(a,b)| = |(c,d)|$.
   b) (5 pt) $|(a,b)| = |[a,b]|$.
   c) (5 pt) $|(a,b)| = |\mathbb{R}|$.

2. Let A be a finite set with $n \geq 1$ elements and B be a finite set with $m \geq 1$ elements.

   a) (5 pt) How many functions are there from A to B?
   b) (5 pt) How many one to one functions are there from A to B?
   c) (5 pt) How many onto functions are there from A to B?

3. (5 pt) Let A be a (possibly infinite) set with $|A| \geq 2$ and let $B = \{f | f : A \longrightarrow A\}$ be the set of functions from A to A. Show that $|A| < |B|$.

4. Consider the set, I, of all integrable functions on the interval $[-1, 1]$. We say that two functions in I, f and g, are equivalent ($f \sim g$) if $\int_{-1}^{1} (f(x) - g(x))\, dx = 0$.

   a) (5 pt) Show that $\sim$ defines an equivalence relation on I.
   b) (5 pt) Find a representative for each equivalence class in $I/\sim$.
   c) (5 pt) Show that $|I/\sim| = |\mathbb{R}|$.

5. (5 pt) Let $(S, \leq)$ be a finite partially ordered set. Show that $(S, \leq)$ is well-ordered if and only if it is totally ordered. Is this statement also true for infinite sets (prove or give a counterexample)?