MATH 270
SUMMER 2007
EXAM 2

1. (5 pt) Let X be a nonempty set and let S_X be the set of bijective functions $f : X \longrightarrow X$. Show that the set S_X forms a group under the operation of function composition.

2. Let A be a finite nonempty set.
   a) (5 pt) Show that A is well-ordered if and only if A is totally ordered.
   b) (5 pt) Show that A is well-ordered if and only if every nonempty subset of A has a *maximal* element.

3. Let S be a set and let $P(S)$ be its power set. Let $X, Y \in P(S)$. We say that $X \sim Y$ if $|X| = |Y|$.
   a) (5 pt) Show that $\sim$ is an equivalence relation on $P(S)$.
   b) (5 pt) If $|S| = n < \infty$, find the number of equivalence classes in $P(S)/\sim$.
   c) (5 pt) Again suppose that $|S| = n < \infty$. For each equivalence class in $P(S)/\sim$, find the number of elements in that equivalence class.
   d) (5 pt) Show that $\sum_{k=0}^{n} \binom{n}{k} = 2^n$.
   e) (5 pt) If S is denumerable, find the number of equivalence classes in $P(S)/\sim$.
   f) (5 pt) If S is denumerable, show that there is an equivalence class containing uncountably many elements of $P(S)$ (for a bonus can you show that there is *exactly* one such class and tell me which one it is?).