MATH 420-620
FALL 2012
HOMEWORK 1

Due Friday August 31, 2012.

1. Let $k, m, n \in \mathbb{Z}$ be nonzero integers.
   a) (5 pt) Show that $\gcd(m, n)$ is a linear combination of m and n (that is, show that there are integers a and b such that $am + bn = \gcd(m, n)$).
   b) (5 pt) Show that if $\gcd(k, m) = 1$ and $\gcd(k, n) = 1$, then $\gcd(k, mn) = 1$.
   c) (5 pt) Show that if $\gcd(k, m) = 1$ and k divides mn, then k divides n.

2. (5 pt) Let $m, n \in \mathbb{Z}$ be nonzero integers, $d = \gcd(m, n)$ and $L = \operatorname{lcm}(m, n)$. Show that $dL = mn$.

3. Let A be a nonempty set and $\sim$ an equivalence relation on A.
   a) (5 pt) Show that the set of equivalence classes of A under $\sim$ is a partition of A.
   b) (5 pt) Show that if $\{A_i\}_{i\in\Lambda}$ is a partition of A, then there is an equivalence relation on A such that the sets $\{A_i\}_{i\in\Lambda}$ are precisely the equivalence classes of A under this relation.