

MATH 420-620
FALL 2012
HOMEWORK 10

Due Wednesday November 7, 2012.

1. (5 pt) Consider the finite abelian group

$$\mathbb{Z}_{108} \oplus \mathbb{Z}_{24} \oplus \mathbb{Z}_{1125} \oplus \mathbb{Z}_{420} \oplus \mathbb{Z}_{620}.$$

- a) (5 pt) Find the invariant factor decomposition for this group.
b) (5 pt) Find the elementary divisor decomposition for this group.
2. (5 pt) Let K be the group of order 2 and H be abelian. Suppose that $\phi : K \rightarrow \text{Aut}(H)$ takes the nonidentity element to the automorphism of H that takes each element to its inverse.
- a) (5 pt) Find necessary and sufficient conditions on H so that $H \rtimes_{\phi} K \cong H \times K$.
b) (5 pt) What can you say about $H \rtimes_{\phi} K$ in the case where H is cyclic?
c) (5 pt) Find all groups of order 8 that cannot be written as the semidirect product of two of its proper subgroups.
3. Show that S_n is not solvable if $n \geq 5$ (you may use the fact that A_n is simple if $n \geq 5$).