MATH 420-620
FALL 2012
HOMEWORK 12

Due Wednesday November 21, 2012.

1. Let R be a commutative ring with identity and let $x, y \in R$ be nilpotent elements.
   a) (5 pt) Show that $x + y$ and xy are nilpotent elements.
   b) (5 pt) Show that if u is a unit of R and x is nilpotent, then $u + x$ is a unit.
   c) (5 pt) Show that if R is not commutative, neither of the above necessarily holds ($x + y$ is not necessarily nilpotent and $u + x$ is not necessarily a unit).

2. Let R be a finite commutative ring.
   a) (5 pt) Show that if R contains an element that is not a zero divisor, then R has an identity.
   b) (5 pt) Explain why every element of R is either a zero divisor or a unit.
   c) (5 pt) Show that any finite integral domain is a field.