

MATH 420-620
FALL 2012
HOMEWORK 2

Due Monday September 10, 2012.

1. (5 pt) Let G be a finite group. Show that any element of G has finite order.
2. (5 pt) Show that any group of exponent 2 is abelian.
3. The goal of this problem is to show that any *finite* group generated by two elements of order two is dihedral (with $\mathbb{Z}_2 \times \mathbb{Z}_2$ being considered a “degenerate” dihedral group). Suppose that G is generated by the elements $x, y \in G$, both of order 2.
 - a) (5 pt) Assuming that the order of G is finite, what can you say about the order of the element $xy \in G$?
 - b) (5 pt) Show that the group generated by x and y is the same as the group generated by xy and y
 - c) (5 pt) Show that the group generated by x and y is dihedral.
4. (5 pt) Let G be a group. Recall that the center of G is defined by $Z(G) = \{z \in G \mid zg = gz, \forall g \in G\}$. Compute $Z(D_n)$.