MATH 420-620
FALL 2012
HOMEWORK 3

Due Monday September 17, 2012.

1. (5 pt) Consider the map $\phi : G \longrightarrow G$ given by $\phi(x) = x^{-1}$.
   a) (5 pt) Show that ϕ is a homomorphism if and only if G is abelian.
   b) (5 pt) Show that if ϕ is a homomorphism, then ϕ is an automorphism.

2. Consider the group, G, generated by the two matrices

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

   a) (5 pt) Find the order of G.
   b) (5 pt) Is G isomorphic to the quaternion group Q_8? Why or why not?

3. Let G be a group. We define $\text{Aut}(G) = \{\phi : G \longrightarrow G |\ \phi \text{ is an automorphism.}\}$.
   a) (5 pt) Show that $\text{Aut}(G)$ is a group.
   b) (5 pt) Suppose we define $\phi_g : G \longrightarrow G$ by $\phi_g(x) = gxg^{-1}$. Show that $\phi_g \in \text{Aut}(G)$.
   c) (5 pt) Consider the collection of all $\phi_g,\ g \in G$ (we call this collection $\text{Inn}(G)$). Show that $\text{Inn}(G)$ is a subgroup of $\text{Aut}(G)$. Is it a normal subgroup of $\text{Aut}(G)$?
   d) (5 pt) Show that the correspondence $g \longrightarrow \phi_g$ is a homomorphism from G to $\text{Aut}(G)$. What is its kernal?