MATH 420-620
FALL 2012
HOMEWORK 5

Due Monday October 1, 2012.

1. Consider the additive group of the rationals $\mathbb{Q}$.
   a) (5 pt) Show that any finitely generated subgroup of $\mathbb{Q}$ is cyclic.
   b) (5 pt) Show that $\mathbb{Q}$ is not finitely generated.

2. (5 pt) Let H and K normal subgroups of G such that $H \bigcap K = 1$. Show that $hk = kh$ for all $h \in H$ and $k \in K$.

3. (5 pt) Classify all groups of order $2p$ where p is an odd prime.

4. (5 pt) Show that if G is a finite abelian group of order greater than 2, then $\mathrm{Aut}(G)$ is a finite group of even order.

5. Suppose that G is a finite group and $N \trianglelefteq G$.
   a) (5 pt) Show that if H is a subgroup of G such that $\gcd(|H|, [G : N]) = 1$ then H is a subgroup of N.
   b) (5 pt) Show that if $\gcd(|N|, [G : N]) = 1$ then N is the unique subgroup of G of order $|N|$.