

**MATH 420-620**  
**FALL 2012**  
**HOMEWORK 6**

1. (5 pt) Show that there is no permutation in  $S_n$  that is both even and odd.
2. (5 pt) Use the information from the first problem to explain why there is no rearrangement on the Rubik's Cube that swaps two corners and leaves all other pieces in their correct (geographic) position.
3. (5 pt) Show that there is an epimorphism

$$\phi : S_n \longrightarrow \mathbb{Z}/2\mathbb{Z}$$

and describe its kernel.

4. In this problem, we will count elements of various orders in  $S_n$ 
  - a) (5 pt) Find all  $k$  such that there is an element of order  $k$  in  $S_5$ , and for each such  $k$  determine how many elements of order  $k$  that there are.
  - b) (5 pt) Let  $p$  be a positive prime. How many elements of order  $p$  are there in  $S_p$ ?
  - c) (5 pt) How many subgroups of order  $p$  are there in  $S_p$ ?