MATH 420-620
FALL 2012
HOMEWORK 7

Due Wednesday, October 17, 2012

1. (5 pt) Let H be a subgroup of G. Show that $\bigcap_{x \in G} xHx^{-1}$ is always normal in G.

2. Let G be a group and H a subgroup. We say that H is characteristic in G if $\phi(H) \subseteq H$ for all $\phi \in \mathrm{Aut}(G)$.
   a) (5 pt) Show that if H is characteristic in G, then H is normal in G.
   b) (5 pt) Show that an arbitrary intersection of characteristic subgroups of G is characteristic.
   c) (5 pt) Give an example of a group G with a normal subgroup that is not characteristic.
   d) (5 pt) Show that $Z(G)$ is a characteristic subgroup of G.

3. (5 pt) Let G be a group and let G act on itself by conjugation, that is, $g \cdot x = gxg^{-1}$.
   a) (5 pt) Show that the above is, in fact, a group action.
   b) (5 pt) What is the kernal of this action?
   c) (5 pt) Show that if $a \in G$ then $G_a = C_G(a)$.

4. (5 pt) Let G be a group. Show that $\mathrm{Inn}(G) \cong G/Z(G)$.