MATH 420-620
FALL 2012
EXAM 1

1. Let G be a group and $x \in G$ and N a subgroup of G.
   a) (3 pt) Define the order of G.
   b) (3 pt) Define the order of x.
   c) (3 pt) Define what it means for N to be normal in G.
   d) (3 pt) What is the centralizer of N in G?
   e) (3 pt) What is the normalizer of N in G?
   f) (3 pt) What is the commutator subgroup of G?

2. (5 pt) Let $H \leq K \leq G$ be groups, and suppose that G is finite. Show that $[G : H] = [G : K][K : H]$.

3. (5 pt) Let G be a group and H be a subgroup of G. Show that the $C_G(H)$ is normal in $N_G(H)$.

4. (5 pt) Let G be a group and H, N normal subgroups of G of finite index such that $[G : H]$ is relatively prime to $[G : N]$. Show that $G = NH$.