MATH 420-620
FALL 2012
EXAM 2

1. Consider the finite abelian group

$$\mathbb{Z}_{243} \oplus \mathbb{Z}_{72} \oplus \mathbb{Z}_{80} \oplus \mathbb{Z}_{150} \oplus \mathbb{Z}_{98}.$$

  a) (5 pt) Find the elementary divisor decomposition of this group.
  b) (5 pt) Find the invariant factor decomposition of this group.

2. (5 pt) Show that there is no simple group of order 108.

3. (5 pt) Classify all abelian groups of order 108.

4. (5 pt) Let G be a simple group of order 168. How many elements of order 7 are there in G?

5. Let G be a group, N a normal subgroup of G, and H a subgroup of G such that $N \subseteq H$. Suppose further that G/N is an abelian group.

  a) (3 pt) What is the relationship between N and G', the commutator subgroup of G?
  b) (3 pt) Prove that H/N is an abelian group.
  b) (3 pt) Prove that H is normal in G.

6. Suppose that G is a solvable group.

  a) (3 pt) If $G^{(n)}$ is the n^{th} commutator subgroup of G, what can you say about the sequence $\{G^{(n)}\}_{n \geq 1}$?
  b) (3 pt) Show that if H is a subgroup of G, then H is also solvable.