MATH 420-620
FALL 2012
FINAL EXAM

1. (5 pt) Classify all abelian groups of order p^2q^3 where p, q are distinct positive primes.

2. (5 pt) Show that there is no simple group of order 50.

3. (5 pt) Let G be a group and G' be its commutator subgroup. What can you say about the commutator subgroup of G/G'?

4. (5 pt) Let R be commutative with identity and $I \subseteq R$ an ideal such that

$$I := \bigcap_{i \in \Gamma} \mathfrak{P}_i$$

where each $\mathfrak{P}_i$ is a prime ideal. Show that I is a radical ideal.

5. (5 pt) Let $d < 0$ be a square free integer. Find the group of units of the domain $\mathbb{Z}[\sqrt{d}]$.

6. (5 pt) Show that the group of units of $\mathbb{Z}[\sqrt{2}]$ is infinite.

7. (5 pt) Let R be a commutative ring with identity. We define the Jacobson radical of R, $J(R)$, to be the intersection of all of the maximal ideals of R. Show that $x \in J(R)$ if and only if $1+rx \in U(R)$ for all $r \in R$.

8. Give the definition for each of the following.
   a) (3 pt) What is an integral domain?
   b) (3 pt) If $H \subseteq G$ is a group, what is the normalizer of H is G?
   c) (3 pt) What is a prime ideal?
   d) (3 pt) What is a maximal ideal?
   e) (3 pt) What is a multiplicatively closed subset of a ring?
   f) (3 pt) What does it mean for a multiplicatively closed set to be saturated?