MATH 421-621
SPRING 2013
HOMEWORK 10

Due Wednesday April 24, 2013.

1. (5 pt) Let $K \subseteq F$ be an extension of fields with F algebraic over K and D an integral domain with $K \subseteq D \subseteq F$. Show that D is a field.

2. (5 pt) Let $K \subseteq F$ be fields. If $u \in F$ is algebraic over K of odd degree, then $K(u) = K(u^2)$.

3. (5 pt) Using the notation of the previous problem, show that if u is transcendental over K, then $K(u^2)$ is a proper subfield of $K(u)$ that is isomorphic to $K(u)$.

4. Let $K \subseteq F$ be fields. We define $\mathrm{Aut}_K(F)$ to be the group of automorphisms of F that fix K. (This is called the Galois group of F over K.) Compute the following Galois groups.

a) (5 pt) $\mathrm{Aut}_{\mathbb{Q}}(\mathbb{Q}(\sqrt{d})$ where d is a square free integer.
b) (5 pt) $\mathrm{Aut}_{\mathbb{Q}}(\mathbb{Q}(\sqrt[3]{2})$.
c) (5 pt) $\mathrm{Aut}_{\mathbb{Q}}(\mathbb{Q}(\sqrt{2}, \sqrt{3})$.
d) (5 pt) $\mathrm{Aut}_{\mathbb{Q}}(\mathbb{R})$.