MATH 421-621
FALL 2013
HOMEWORK 5

Due Wednesday, February 13, 2013.

1. Consider the following commutative diagram of $R-$module homomorphisms with exact rows.

$$\begin{array}{ccccccccc} A_1 & \xrightarrow{f_1} & A_2 & \xrightarrow{f_2} & A_3 & \xrightarrow{f_3} & A_4 & \xrightarrow{f_4} & A_5 \\ \downarrow{\scriptstyle g_1} & & \downarrow{\scriptstyle g_2} & & \downarrow{\scriptstyle g_3} & & \downarrow{\scriptstyle g_4} & & \downarrow{\scriptstyle g_5} \\ B_1 & \xrightarrow{h_1} & B_2 & \xrightarrow{h_2} & B_3 & \xrightarrow{h_3} & B_4 & \xrightarrow{h_4} & B_5 \end{array}$$

Show the following.

a) (5 pt) If g_2 and g_4 are onto and g_5 is one to one then g_3 is onto.
b) (5 pt) If g_2 and g_4 are one to one and g_1 is onto then g_3 is one to one.
c) (5 pt) Explain how the Short Five Lemma follows from the above results.

2. (5 pt) Consider the following short exact sequence of $R-$modules.

$$0 \longrightarrow A \xrightarrow{f} B \xrightarrow{g} C \longrightarrow 0$$

Show that the following two conditions are equivalent:

i) There is an $R-$module homomorphism $k : B \longrightarrow A$ such that $kf = 1_A$.
ii) There is an $R-$module homomorphism $h : C \longrightarrow B$ such that $gh = 1_C$.

3. (5 pt) Show that the $R-$module P is projective if and only every short exact sequence of the form

$$0 \longrightarrow A \xrightarrow{f} B \xrightarrow{g} P \longrightarrow 0$$

is split exact.