# MATH 421-621
# FALL 2013
# HOMEWORK 6

*Due Wednesday, February 27, 2013.*

1. We say that the $n \times n$ matrix $A$ is nilpotent if $A^m = 0$ for some positive integer $m$.
   a) (5 pt) Show that every eigenvalue of a nilpotent matrix is 0.
   b) (5 pt) Show that if $A$ is nilpotent and $n \times n$, then $A^n = 0$.

2. Recall that $A$ and $B$ are similar $n \times n$ if there is a nonsingular $n \times n$ (say $P$) such that $B = P^{-1}AP$.
   a) (5 pt) Show that $\det(A) = \det(B)$.
   b) (5 pt) Show that if $(\lambda, v)$ is an eigenvalue/eigenvector pair for the matrix $A$ then $\lambda$ is an eigenvalue for $B$ (corresponding to what eigenvector of $B$?).

3. (5 pt) Show that the $n \times n$ matrix $A$ is similar to a diagonal matrix if and only if $A$ has $n$ linearly independent eigenvectors.

4. (5 pt) Let $V$ be an $n-$dimensional vector space over $\mathbb{F}$ and $\phi : V \longrightarrow V$ a fixed linear transformation. Show that $V$ is a $\mathbb{F}[x]-$module with action defined by

$$f(x) \cdot v = f(\phi)(v)$$

where $f(x) \in \mathbb{F}[x]$ and $v \in V$.