MATH 421-621
SPRING 2013
EXAM 1

1. Give definitions/brief answers for the following.
   a) (3 pt) Define the notion of an $R-$module.
   b) (3 pt) What is an $R-$module homomorphism?
   c) (3 pt) What does it mean for an $R-$module to be free?
   d) (3 pt) What does it mean for an $R-$module to be projective?
   e) (3 pt) What does it mean for a sequence of $R-$module homomorphisms to be exact?
   f) (3 pt) What is a short exact sequence of $R-$module homomorphisms?
   g) (3 pt) What does it mean for a short exact sequence of $R-$module homomorphisms to split?
   h) (3 pt) If V is a vector space over $\mathbb{F}$, what is the definition of the dual vector space V^*?
   i) (3 pt) If V is a vector space over the field $\mathbb{F}$, give condition(s) which guararantee that $V \cong V^{**}$.

2. (5 pt) Let M be an $R-$module. Show that any element of the $R-$module $R \otimes_R M$ can be written as a *single* tensor of the form $1_R \otimes m$ with $m \in M$.

3. (5 pt) Compute the $\mathbb{Z}-$module $\mathbb{Q} \otimes_{\mathbb{Z}} A$, where A is an abelian group of order n.

4. (5 pt) Let $0 \longrightarrow V_1 \longrightarrow V_2 \longrightarrow V_3 \longrightarrow 0$, be a short exact sequence of vector spaces (over $\mathbb{F}$). Show this sequence splits.

5. Let P and Q be projective $R-$modules.
   a) (5 pt) Show that $P \oplus Q$ is projective.
   b) (5 pt) If $P \cong A \oplus B$ where A and B are $R-$modules, then show that A and B must be projective.