MATH 720
FALL 2010
HOMEWORK 1

Due Friday September 3, 2010.

1. Let $k, m, n \in \mathbb{Z}$ be nonzero integers.
   a) (5 pt) Show that $\gcd(m, n)$ is a linear combination of m and n (that is, show that there are integers a and b such that $am + bn = \gcd(m, n)$).
   b) (5 pt) Show that if $\gcd(k, m) = 1$ and $\gcd(k, n) = 1$, then $\gcd(k, mn) = 1$.
   c) (5 pt) Show that if $\gcd(k, m) = 1$ and k divides mn, then k divides n.

2. (*2 is, in fact, odd*) Let G be a group.
   a) (5 pt) Show that any group of exponent 2 is abelian.
   b) (5 pt) Show that if G is finite and generated by two elements of order 2, then $G \cong D_n$ for some n.

3. (5 pt) Let S be a semigroup. Show that S is a group if and only if S has a left identity and every element of S has a left inverse.

4. (5 pt) Show that the group G is abelian if and only if the function $\phi : G \longrightarrow G$ given by $\phi(x) = x^{-1}$ is an automorphism.

5. (5 pt) Let G be a group and $\mathrm{Aut}(G) = \{\phi : G \longrightarrow G | \phi \text{ is an automorphism}\}$. If $x \in G$, we define the function $\phi_x : G \longrightarrow G$ by $\phi_x(y) = x^{-1}yx$ for all $y \in G$, and we define $\mathrm{Inn}(G) = \{\phi_x | x \in G\}$. Show that $\mathrm{Aut}(G)$ is a group with subgroup $\mathrm{Inn}(G)$.