MATH 720
FALL 2003
HOMEWORK 3

Due Monday September 29, 2003.

1. (5 pt)Suppose that G and H are groups and $f : G \longrightarrow H$ and $g : H \longrightarrow G$ are 1-1 group homomorphisms. Does is follow that $G \cong H$? Prove or give a counterexample.

2. We define the following generalizations of S_n, namely $S_\infty = \{f : \mathbb{N} \longrightarrow \mathbb{N} | f \text{ is bijective}\}$ and $\overline{S} = \bigcup_{n=1}^{\infty} S_n$.
   a) (5 pt) Show that both S_∞ and $\overline{S}$ are groups.
   b) (5 pt) Show that $\overline{S} \subseteq S_\infty$.
   c) (5 pt) Show that $\overline{S}$ and S_∞ are not isomorphic.

3. Show that S_n can be generated by the following sets of elements:
   a) (5 pt) $\{(1\ 2), (1\ 2 \cdots n)\}$.
   b) (5 pt) $\{(1\ 2), (1\ 3), \cdots, (1\ n)\}$.
   c) (5 pt) $\{(1\ 2), (2\ 3), \cdots, (n-1\ n)\}$.

4. (5 pt) Let D_n be the dihedral group on the $n-$gon. Compute $Z(D_n)$ and find all normal subgroups of D_n.

5. For this problem we consider the alternating and symmetric groups.
   a) (5 pt) Show that A_n is the unique subgroup of S_n of index 2.
   b) (5 pt) Show that the only nontrivial normal subgroup of S_n $(n \neq 4)$ is A_n.

6. (5 pt) Prove that neither S_n nor Q_8 can be decomposed into a direct product of proper subgroups.