MATH 720
FALL 2010
HOMEWORK 3

Due Friday October 1, 2010.

1. (5 pt) Give an example to show that the notions of direct product and direct sum are distinct (that is, give a family of groups such that $\oplus G_i \not\cong \prod G_i$).

2. (5 pt) Suppose that G and H are groups and $\phi : G \longrightarrow H$ and $\psi : H \longrightarrow G$ are both injective homomorphisms. Does this imply that $G \cong H$?

3. We define the following generalizations of S_n. Let $S_\infty = \{f : \mathbb{N} \longrightarrow \mathbb{N} | f \text{ is bijective}\}$ and $\overline{S} = \bigcup_{n=1}^{\infty} S_n$.

   a) (5 pt) Show that S_∞ is a group with subgroup $\overline{S}$.

   b) (5 pt) Show that S_∞ and $\overline{S}$ are not isomorphic.

4. (5 pt) Show that S_n can be generated by the following sets.

   a) (5 pt) $\{(1\ 2), (1\ 2 \cdots n)\}$.

   b) (5 pt) $\{(1\ 2), (1\ 3), \cdots, (1\ n)\}$.

5. For this problem, we consider the groups $A_n \trianglelefteq S_n$.

   a) (5 pt) Show that A_n is the unique subgroup of S_n of index 2.

   b) (5 pt) Show that for all $n \neq 4$, A_n is the unique normal subgroup of S_n.