MATH 720
FALL 2003
HOMEWORK 4

Due Monday October 13, 2003.

1. (5 pt) Let G be a finite abelian group. Show that if G is not cyclic, then G contains an isomorphic copy of $\mathbb{Z}_p \oplus \mathbb{Z}_p$.

2. (5 pt) Show that every finitely-generated subgroup of $\mathbb{Q}$ is cyclic.

3. (5 pt) Let F be a free group on the set X. Consider some fixed integer n and show that the subgroup generated by the set $\{g^n | g \in G\}$ is a normal subgroup of F.

4. (5 pt) Let G be a group. Suppose that there is an element $a \in G$ such that a has precisely two distinct conjugates in G. Show that G cannot be simple.

5. (5 pt) Let G be a group. Show that $\mathrm{Inn}(G) \cong G/Z(G)$.