

**MATH 720**  
**FALL 2010**  
**HOMEWORK 4**

*Due Monday October 23, 2010.*

1. (5 pt) Let  $G$  be a group and  $N$  a normal subgroup of  $G$ . Suppose that  $H$  is a subgroup of  $G$  such that  $NH = G$  and  $N \cap H = e$ . Show that  $G$  is isomorphic to the semidirect product of  $N$  and  $H$ ,
2. (5 pt) Show that if  $|G| > 2$  then  $\text{Aut}(G)$  is nontrivial.
3. (5 pt) Let  $p$  be a nonzero prime. Show that if  $|G| = pn$  with  $p > n$ . Show that  $G$  has a normal subgroup of order  $p$ .
4. Let  $p$  and  $q$  be distinct primes. Show that there are no simple groups of order:
  - a) (5 pt)  $pq$ ,
  - b) (5 pt)  $p^2q$ ,
  - c) (5 pt)  $56$ ,
  - d) (5 pt)  $2^3 3^k$ ,  $k \geq 1$ ,
  - e) (5 pt)  $80$ .