MATH 720
FALL 2010
HOMEWORK 6

Due Friday, November 12, 2010.

1. (5 pt) Let R be a finite ring. Show that if R has an element that is not a zero divisor, then R has an identity. Conclude that if R is any finite ring, then every element of R is either a zero divisor or a unit.

2. (5 pt) We say that a Boolean ring is a ring such that $x^2 = x$ for all $x \in R$. Show that a Boolean ring is commutative and of characteristic 2.

3. (5 pt) Show that if R is commutative, then the set of nilpotent elements is an ideal (and show that this is not true in the noncommutative case).

4. Let R be a nonzero ring such that for all $0 \neq a \in R$, there is a unique $b \in R$ such that $aba = a$.

   a) (5 pt) Show that R has no nontrivial zero divisors.
   b) (5 pt) With the notation as above, show that $bab = b$.
   c) (5 pt) Show that R has an identity.
   d) (5 pt) Show that R is a division ring.

5. (5 pt) Suppose R is commutative of prime characteristic $p > 0$. Show that the function

$$\phi_n : R \longrightarrow R$$

given by $\phi_n(x) = x^{p^n}$ is a ring homomorphism.