MATH 720
FALL 2003
EXAM 1

Due Monday, October 20, 2003.

1. Let G be a group with center $Z(G)$.
   a) (5 pt) Show that if $G/Z(G)$ is cyclic, then G is abelian.
   b) (5 pt) Use this result to show that if $|G| = p^2$ with p a positive prime integer, then G is abelian.
   c) (5 pt) Show that if $|G| = p^3$ then

$$Z(G) \cong \begin{cases} G & \text{if } G \text{ is abelian} \\ \mathbb{Z}_p & \text{if } G \text{ is not abelian} \end{cases}$$

   d) (5 pt) Show that if $|G| = p^3$ and G is not abelian, then $G/Z(G) \cong \mathbb{Z}_p \oplus \mathbb{Z}_p$.

2. (5 pt) Let p, q, r be distinct positive prime integers. Show that there is no simple group of order pqr.

3. Let p and q be distinct positive prime integers.
   a) (5 pt) Show that there is no simple group of order p^n, $n > 1$.
   b) (5 pt) Show that there is no simple group of order p^2q.

4. Prove the following statements for groups of specific order.
   a) (5 pt) Show that any group of order 35 is cyclic.
   b) (5 pt) Show that any group of order 99 is abelian and classify them all.
   c) (5 pt) Show that no group of order 24 is simple.
   d) (5 pt) Show that no group of order 72 is simple.