

**MATH 721
SPRING 2011
HOMEWORK 4**

Due Friday March 25, 2011.

1. Find the canonical forms (the rational canonical form, primary rational canonical form and Jordan canonical form if possible) for the following matrices over \mathbb{Q} :

a) (5 pt)
$$\begin{bmatrix} 3 & 1 & 0 & 1 & 1 \\ 0 & 3 & 0 & -1 & 0 \\ 0 & -2 & 4 & 2 & 0 \\ 0 & -1 & 0 & 3 & 0 \\ 1 & -1 & 0 & -1 & 3 \end{bmatrix}$$

b) (5 pt)
$$\begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 1 & -2 & -1 & 0 & 0 \\ -5 & 8 & 3 & 0 & 0 \\ 15 & -31 & -9 & 3 & 1 \\ -40 & 82 & 21 & -9 & -3 \end{bmatrix}$$

2. For the following fields, F , find the group of \mathbb{Q} -automorphisms of F .

- a) (5 pt) $F = \mathbb{Q}(i)$
b) (5 pt) $F = \mathbb{Q}(\sqrt[3]{2})$
c) (5 pt) $F = \mathbb{R}$.

3. (5 pt) Let $K \subseteq D \subseteq F$ with K, F fields with F algebraic over K and D an integral domain. Show that D is a field.