MATH 721
SPRING 2004
HOMEWORK 6

Due Friday April 23, 2004.

1. (5 pt) Let F be a finite field. Show that any element in F can be written as the sum of two squares.

2. Let K be a finite field and F an algebraic closure of K.

   a) (5 pt) Show that $\mathrm{Gal}(F/K)$ is abelian.

   b) (5 pt) Show that every nonidentity element of $\mathrm{Gal}(F/K)$ is of infinite order.

3. (5 pt) Let K be a field of characteristic 0. Show that any extension field of K is seperable.

4. (5 pt) $K \cong \mathbb{Z}_p$ and $f \in K[x]$ is irreducible then show f divides $x^{p^n} - x$ if and only if $\deg(f)$ divides n.

5. Let the characteristic of K be $p > 0$.

   a) (5 pt) If $[F : K] = n$ and p does not divide n, then F is separable over K.

   b) (5 pt) If $u \in F$ is algebraic over K, then u is separable over K if and only if $K(u) = K(u^{p^n})$ for all $n \geq 1$.

6. (5 pt) Let F be an extension of K and $u, v \in F$ such that u is separable over K and v is totally inseperable over K. Show that $K(u + v) = K(u, v)$. Also show that $K(u, v) = K(uv)$ if both u and v are nonzero.