

**MATH 721
SPRING 2011
HOMEWORK 6**

Due Friday May 6, 2011.

1. (5 pt) Let F be a finite field. Show that any element in F can be written as the sum of two squares.
2. Let K be a finite field and F an algebraic closure of K .
 - a) (5 pt) Show that $\text{Gal}(F/K)$ is abelian.
 - b) (5 pt) Show that every nonidentity element of $\text{Gal}(F/K)$ is of infinite order.
3. Let the characteristic of K be $p > 0$.
 - a) (5 pt) If $[F : K] = n$ and p does not divide n , then F is separable over K .
 - b) (5 pt) If $u \in F$ is algebraic over K , then u is separable over K if and only if $K(u) = K(u^{p^n})$ for all $n \geq 1$.
4. (5 pt) Let F be an extension of K and $u, v \in F$ such that u is separable over K and v is totally inseparable over K . Show that $K(u + v) = K(u, v)$. Also show that $K(u, v) = K(uv)$ if both u and v are nonzero.