MATH 721
SPRING 2004
FINAL EXAM

Due Friday May 14, 2004. As is usual on exams, I am the only biological resource that you should use.

1. (5 pt) Show that any invertible ideal in a quasilocal domain is principal.

2. (5 pt) Let V be a valuation domain and V' an overring of V (that is, V' is a ring between V and its quotient field). Show that V' is a valuation domain.

3. (5 pt) Let R be an integral domain with quotient field K. Show that the integral closure of R (which we will call $\overline{R}$) is

$$\overline{R} = \bigcap_{R \subseteq V \subseteq K} V$$

where each V is a valuation overring of R.

4. (5 pt) Show that any Dedekind domain with only finitely many prime ideals is necesarily a PID.

5. (5 pt) Show that if I is an invertible ideal of the domain R, and S is a multiplicatively closed set ($0 \notin S$) then $S^{-1}I$ is invertible in R_S. Use this to show that any localization of a Dedekind domain is a Dedekind domain.