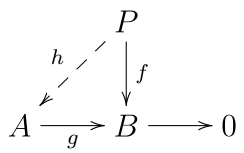
MATH 724
SUMMER 2009
HOMEWORK 1

Due Sometime.

1. (5 pt) Show that if $T_1 : \mathfrak{A} \longrightarrow \mathfrak{B}$ and $T_2 : \mathfrak{B} \longrightarrow \mathfrak{C}$ are both covariant or both contravariant (functors), then $T_2 \circ T_1$ is covariant.

2. Give examples for each of the following.
   a) (5 pt) Show that $\text{Hom}_R(\prod_{\alpha \in I} A_\alpha, B)$ is not generally isomorphic to $\prod_{\alpha \in I} \text{Hom}_R(A_\alpha, B)$.
   b) (5 pt) Show that $\text{Hom}_R(\prod_{\alpha \in I} A_\alpha, B)$ is not generally isomorphic to $\oplus_{\alpha \in I} \text{Hom}_R(A_\alpha, B)$.
   a) (5 pt) Show that $\text{Hom}_R(B, \oplus_{\alpha \in I} A_\alpha)$ is not generally isomorphic to $\oplus_{\alpha \in I} \text{Hom}_R(B, A_\alpha)$.
   a) (5 pt) Show that $\text{Hom}_R(B, \oplus_{\alpha \in I} A_\alpha)$ is not generally isomorphic to $\prod_{\alpha \in I} \text{Hom}_R(B, A_\alpha)$.

3. (20 pt) Given the diagram of $R-$modules below with exact bottom row,



we say that P is projective if there is an $R-$module homomorphism $h : P \longrightarrow A$ such that $gh = f$.

Prove the following conditions are equivalent:
   a) P is projective.
   b) Every short exact sequence of the form $0 \longrightarrow A \longrightarrow B \longrightarrow P \longrightarrow 0$ is split exact.
   c) There is an $R-$module K and a free module F such that $F \cong P \oplus K$.
   d) $\text{Hom}_R(P, -)$ is an exact functor.

4. (20 pt) Dualize the previous problem to "invent" injective modules and prove the equivalence of the analogous conditions.