MATH 724
SUMMER 2010
HOMEWORK 1

Due Monday, June 28, 2010.

1. Let V be a valuation domain. Prove the following.
   a) (5 pts) Show that V is integrally closed.
   b) (5 pts) Show that every overring of V is a valuation domain.
   c) (5 pts) Show that every overring of V is a localization of V.

2. (5 pts) Let $\mathbb{F}$ be a field. Find an overring of $\mathbb{F}[x, y]$ that is not integrally closed or prove that no such overring can exist.

3. Let R be an integral domain, $I \subseteq R$ a nonzero ideal, and S a multiplicatively closed subset of R.
   a) (5 pts) Show that if I is an invertible ideal of R then I_S is an invertible ideal of R_S.
   b) (5 pts) Show that if I is finitely generated then I is an invertible ideal of R if and only if $I_{\mathfrak{M}}$ is principal for all $\mathfrak{M} \in \mathrm{MaxSpec}(R)$.