MATH 724
FALL 2005
HOMEWORK 2

Due Monday September 19, 2005.

1. A ring is called *Von Neumann regular* if for every $x \in R$ there is a $y \in R$ such that $xyx = x$. Let R be a (commutative) Von Neumann regular ring.

a) (5 pt) Show that if R is an integral domain, then R is a field.
b) (5 pt) Show that any direct product of fields is Von Neumann regular.
c) (5 pt) Show that if $\mathfrak{P}$ is a prime ideal in R then $R_{\mathfrak{P}} \cong R/\mathfrak{P}$.

2. A ring is called Artinian if it satifies the descending chain on prime ideals (that is given any descending chain of ideals $I_1 \supseteq I_2 \supseteq I_3 \cdots$, there is a natural number N such that $I_n = I_N$ for all $n \geq N$).

a) (5 pt) Show that any Artinian domain is a field.
b) (5 pt) Show that R is Artinian if and only if R is Noetherian and zero-dimensional.

3. Let R be an integral domain and consider the ring of formal power series $R[[x]]$.

a) (5 pt) Show that $U(R[[x]]) = \{f(x) \in R[[x]] | f(0) \in U(R)\}$.
b) (5 pt) Show that there is a one-to-one correspondence between the maximal ideals of R and the maximal ideals of $R[[x]]$.
c) (5 pt) Show that if R is a PID, then $R[[x]]$ is a UFD.