

MATH 724
FALL 2005
HOMEWORK 4

Due Friday October 21, 2005.

1. (5 pt) Show that the domain V is a valuation domain if and only if all of the ideals of V are linearly ordered (can you replace “ideals” with “prime ideals” here?).
2. (5pt) Show that if the valuation domain, V , is Noetherian then $\dim(V) \leq 1$.
3. (5 pt) Construct a non-Noetherian, 1-dimensional valuation domain.
4. Let V be a valuation domain and x a nonzero element.
 - a) (5 pt) Show that x is irreducible if and only if x is prime.
 - b) (5 pt) If the valuation domain, V , is not a field, then show that V contains a nonzero prime element if and only if the maximal ideal of V is principal.
 - c) (5 pt) Show that V is atomic if and only if V is a UFD.
5. (5 pt) Given a valuation domain, V , construct the complete integral closure of V (hint: there should be two cases).