MATH 724
SUMMER 2010
HOMEWORK 4

Due Friday, August 6, 2010.

1. (5 pt) For this first one, you may use the fact that is I is a fractional ideal, then there are elements $x, y \in I$ such that $R : (R : I) = R : (R : (Rx + Ry))$. The problem is to explain why if I is divisorial then there are elements $u, v \in (R : I) \setminus \{0\}$ such that $I = Ru^{-1} \bigcap Rv^{-1}$.

2. Let R be an integral domain with quotient field K. We say that the element $\omega \in K$ is Ω-almost integral if $r\omega \in R$ implies that we can find a positive integer b such that $r^b \omega^n \in R$ for all $n \geq 0$. Show that following.

a) (5 pt) If ω is Ω-almost integral, then ω is almost integral.
b) (5 pt) Give an example of an almost integral element that is not Ω-almost integral.
c) (5 pt) Show that V is a valuation domain, then V is Ω-almost integrally closed.
d) (5 pt) Show that D is a Prüfer domain, then D is Ω-almost integrally closed.