MATH 724
FALL 2005
HOMEWORK 6

Due Wednesday December 7, 2005.

1. (5 pt) Suppose that R is a Dedekind domain with class group isomorphic to $\mathbb{Z}_4$. Additionally, assume that all the prime ideals of R are in the principal class and the class of order 2. Show that R is an HFD.

2. (5 pt) Let $R \subseteq T$ be an extension of rings. Show that any minimal prime ideal of R is lain over by a prime ideal of T (that is, if $\mathfrak{P}$ is a minimal prime ideal of R, then there is a prime ideal $\mathfrak{Q} \subseteq T$ such that $\mathfrak{Q} \bigcap R = \mathfrak{P}$).

3. (5 pt) Show that $R \subseteq R[x]$ is LO and GD, but is never INC. Show that this extension fails to be GU is $\dim(R) \geq 1$.

4. Let R be a domain and I a fractional ideal of R.

a) (5 pt) Show that I is invertible if and only if I is a projective $R-$module.
b) (5 pt) Show that if I is invertible, it is of rank 1 (that is, there is an $R-$module, J, such that $I \otimes_R J \cong R$).
c) (5 pt) Show that if I and J are rank 1 projective $R-$modules, then $I \otimes_R J$ is also a rank 1 projective $R-$ module. Conclude that the set of isomorphism classes of rank 1 projective $R-$modules forms a group with multiplication given by $\otimes_R$ (this group is called the Picard group of R and is denoted by $\mathrm{Pic}(R)$).