MATH 726
SUMMER 2005
HOMEWORK 1

Due Wednesday June 22, 2005.

1. (5 pt) Let M be a monoid. We think of M as a category ($\mathfrak{C}$) by defining $\mathrm{obj}(\mathfrak{C}) = \star$ and $\mathrm{Hom}(\star, \star) = M$. The composition of morphisms is given by the multiplication in M. Find all equivalences in this category.

2. (5 pt) Show that the category of rings has an initial and terminal object, but the category of rings with identity has only an initial object (in the category of rings with identity, we declare that every homomorphism of rings $\phi : R \longrightarrow T$ must take the identity of R to the identity of T).

3. (5 pt) Let $\mathfrak{G}$ be the category of groups and $\mathfrak{A}$ the category of abelian groups. Show that the assignment $G \mapsto G/G'$ (G' is the commutator subgroup of G) defines a covariant functor from $\mathfrak{G}$ to $\mathfrak{A}$.

4. (5 pt) Let R be a commutative ring with identity and M a fixed $R-$module. If D denotes any $R-$module, show that the assignment $D \mapsto \mathrm{Hom}_R(D, M)$ defines a contravariant functor from the category of $R-$ modules to itself.

5. (5 pt) Show that if $T_1 : \mathfrak{A} \longrightarrow \mathfrak{B}$ and $T_2 : \mathfrak{B} \longrightarrow \mathfrak{C}$ are both covariant or both contravariant (functors), then $T_2 \circ T_1$ is covariant.