MATH 726
SUMMER 2005
HOMEWORK 2

Due Tuesday July 5, 2005.

1. Give examples for each of the following.
   a) (5 pt) Show that $\mathrm{Hom}_R(\prod_{\alpha\in I} A_\alpha, B)$ is not generally isomorphic to $\prod_{\alpha\in I}\mathrm{Hom}_R(A_\alpha, B)$.
   b) (5 pt) Show that $\mathrm{Hom}_R(\prod_{\alpha\in I} A_\alpha, B)$ is not generally isomorphic to $\oplus_{\alpha\in I}\mathrm{Hom}_R(A_\alpha, B)$.
   a) (5 pt) Show that $\mathrm{Hom}_R(B, \oplus_{\alpha\in I} A_\alpha)$ is not generally isomorphic to $\oplus_{\alpha\in I}\mathrm{Hom}_R(B, A_\alpha)$.
   a) (5 pt) Show that $\mathrm{Hom}_R(B, \oplus_{\alpha\in I} A_\alpha)$ is not generally isomorphic to $\prod_{\alpha\in I}\mathrm{Hom}_R(B, A_\alpha)$.

2. (5 pt) Show that it is not true in general that $\prod_{\alpha\in I}(A_\alpha \otimes_R B)$ is isomorphic to $(\prod_{\alpha\in I} A_\alpha) \otimes_R B$.

3. (5 pt) Let F be an additive functor from the category of $R-$modules to itself. Show that if the sequence

$$0 \longrightarrow A \longrightarrow B \longrightarrow C \longrightarrow 0$$

is split exact then $F(A)$ is a summand of $F(B)$. Use this to show that F preserves finite sums.

4. (5 pt) Show that $\mathrm{Hom}_R(P, -)$ is an exact functor (from the category of $R-$modules to itself) if and only if P is a projective $R-$module.