**MATH 726**
**SUMMER 2005**
**HOMEWORK 3**

*Due Monday July 18, 2005.*

1. (5pt) If $F$ is an additive, right exact functor that preserves sums, show that $F$ preserves direct limits.

2. (5 pt) Prove that any $R-$module $A$ is the direct limit of its finitely generated submodules.

3. (5 pt) Let $A_1 \subseteq A_2 \subseteq A_3 \subseteq \cdots$ be an ascending chain of sets. Compute $\varinjlim A_i$.

4. (5 pt) Prove that the inverse limit, if it exists, is unique. Then show that if the index set has the trivial quasi-order, then $\varprojlim A_i \cong \prod A_i$.

5. (5 pt) Show that if $p$ is prime then the $\mathbb{Z}-$module, $\mathbb{Z}_p$, is an uncountable principal ideal domain with a unique nonzero prime ideal. Also show that if $I \subseteq \mathbb{Z}_p$ is any nonzero ideal, then $\mathbb{Z}_p/I$ is a finite ring.